An Economic and Game Theory Approach to the Legalisation of Rhino Horn Trade

Mathematics of Industry Study Group Graduate Workshop 2016

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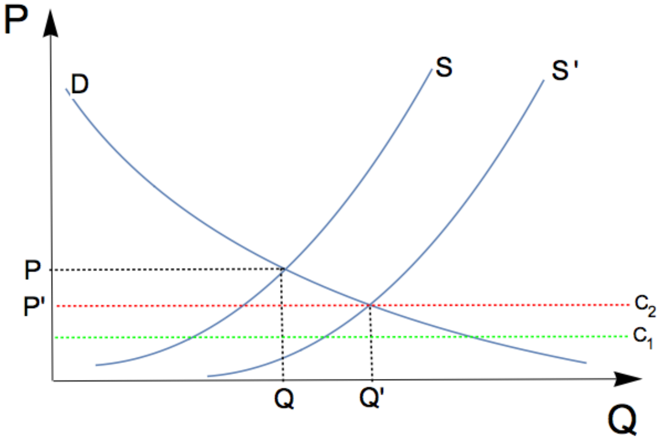
Problem Introduction

- South Africa has a rhino population of close to 20405 white rhinos and 5055 black rhinos.
- In 2007 close to 13 rhinos were poached for their horn. This is in stark contrast with the near 1200 rhinos that were being poached by 2014. At this rate rhinos have less than a decade to survive.
- It is of interest to consider interventions that might allow for the rhino population to last indefinitely.

Problem Introduction

- The South African government has a stockpile of rhino horn.
- It is of interest to consider if this stockpile could be used to manipulate the market for rhino horn in order to deter poachers and save the rhino population.
- Since the stockpile is larger than what it is believed illegal poachers have at their disposal the government would have market power. Naively this suggests a monopoly or oligopoly market.

Problem Introduction



Oligopoly Market

- Oligopoly is a market structure such that multiple firms have market power, the ability to affect prices and market quantity.
- Bowley (1924) introduced conjectural variations (CV) as a solution to the oligopoly problem.
- Conjectural variation takes into account the behaviour that firms expect from their competitors. Their competitors, however, have expectations of the original firm's behavior. Therefore the conjectural variations are responses that other firms have to the original firm's behavior.
- This will allow for modeling the behavior of the government (legal market) and the poachers (illegal market) in simultaneous response to each other.

Model Outline

- Start by considering n separate firms
- By selecting the amount (α) of stock pile (S) that legal firms will put onto the market we should be able to define the optimal strategies for the various players.

Variables

- P is the price per kilo in the market
- q_i is the quantity of horn placed into the market, by firm i. Note that for $i \neq j$ $q_j = f(q_i)$. The derivative of this quantity is the conjectural variation
- $Q = \sum_{i=1}^{n} q_i$ is the total quantity of rhino horn in the market.
- k_i is the cost of producing a kilo of rhino horn, by firm i.

Price

The price at which firms will sell horn's, is determined by the following formula;

$$P(Q) = a - bQ$$

Where a gives the maximum possible price for rhino horn, and b gives the value for which price will decrease, given that one more rhino horn is added to the market.

Profit

The profit Π_i for each firm, is given by the following;

$$\Pi_i = q_i P(Q) - C_i(q_i)$$

where Q can be given by; $\left(q_i + \sum_{j=1;j\neq i}^{n} q_j\right)$

Optimising the Profit

We take the derivative of the profit function, and set this equal to zero;

$$0 = \frac{d\pi_i}{dq_i}$$

Given the conjectural variation, $\frac{dq_j}{dq_i} = r_{ij}$;

$$0 = -q_i b(1 + r_i) + \left(a - b\sum_{j}^{n} q_j\right) - C_i'(q_i)$$

Note that $\sum_{j=1;j\neq i}^{n} r_{ij} = r_i$

2 Firms

In this particular context, we can consider two firms, the illegal firm (the poachers) and the legal firm (the government). We represented the legal firm as firm 1, and the illegal firm as firm 2.

We aim to look at the optimal quantity of legal rhino horns that must be initially released into the market, such that the market price makes it no longer worthwhile to continue poaching.

Determining the Optimum Profit

From the equations given for n firms, we can generalise this to the two firm case, giving the following equations for profit;

$$\Pi_1 = q_1 a - b q_1^2 - b q_1 q_2 - k_1 q_1$$
$$\Pi_2 = q_2 a - b q_1 q_2 - b q_2^2 - k_2 q_2$$

Determining the Optimum Profit

To determine the optimal profit from the previous equations, we take the derivative of each equation with respect to q_1 and q_2 respectively, which gives us the following;

$$\frac{\mathrm{d}\Pi_1}{\mathrm{d}q_1} = a - 2bq_1 - bq_2 - bq_1r_1 - k_1 = 0$$
$$\frac{\mathrm{d}\Pi_2}{\mathrm{d}q_2} = a - bq_1 - 2bq_2 - bq_2r_2 - k_2 = 0$$

Determining the Optimum Profit

Rearranging these equations gives the optimal quantity curves for the legal and illegal sector, which are given by $\overline{q_1}$ and $\overline{q_2}$ respectively.

$$\overline{q_1} = \frac{a - k_1 - bq_2}{2b + br_1}$$
$$\overline{q_2} = \frac{a - k_2 - bq_1}{2b + br_2}$$

Finding the intersection between these two curves gives the optimal quantities (q_1^*, q_2^*) that each firm should add to the market to maximise profit.

$$(q_1^*, q_2^*) = \left(\frac{k_2 + a(1 + r_2) - k_1(2 + r_2)}{b(3 + 2r_2 + r_1(2 + r_2))}, \frac{k_1 + a(1 + r_1) - k_2(2 + r_1)}{b(3 + 2r_2 + r_1(2 + r_2))} \right)$$

Estimating the parameters

If $P = k_2$, which gives the cost of the poachers to produce one kilo of rhino horn, the illegal market doesn't make a profit. This is our aim, as poaching would not be worthwhile if this was the case.

Consequently, to determine the b in our profit function, we set the price to be k_2;

$$P = k_2 = a - b(q_1 + q_2)$$

At the start of legalisation we have a large stockpile (S) available for injection into the market. As well as this, we assume that the illegal market has little to no stockpile readily available. Therefore, $q_1 >> q_2$ and as a result;

$$k_2 = a - bq_1$$

Estimating the parameters

Since we wish to test the effect of adding different percentages of our stockpile into the market initially, we let $q_1 = \alpha S$ where $\alpha \in (0, 1]$. We then find that for an initial injection $q_1 = \alpha S$;

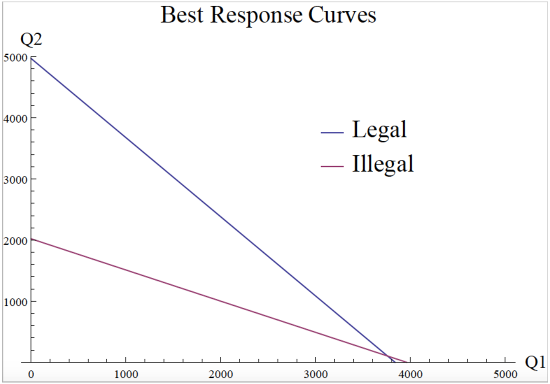
$$b = \frac{a - k_2}{\alpha S}$$

By researching the current illegal market, we can reasonably estimate a, k_1 and b. The conjectural variations r_1 and r_2 are estimated from properties of oligopoly markets.

Results

Using Mathematica, we varied the value of α, and plotted the best response curves. We determined that initially injecting a total of 5% into the market was the most appropriate quantity to drive illegal poachers from the market. The best response curves for this particular system are plotted in the following slide;

Results



The intersection between these best response curves confirms the belief that an initial injection of 5% into the market is the optimal strategy. We anticipate this will correspond to a depletion of illegal poaching.

Limitations of the Static Game

1. The game is not dynamic. Yet the interpretation is.
2. In estimating the parameters assumptions about the illegal stock pile are made $q_1 >> q_2$
3. The conjectural variations (r_{ij}) are not determined in the game. This suggests that additional analysis is required in order to find the global optimal solution.
4. Commitment to a specific strategy (open-loop game)

Introducing Time

The Dynamical System

Considering our Price-equation we now introduce a time-variable to our Quantity, that is $Q = Q(t)$.
We then obtain

$$P(t) = a - bQ(t)$$

where

$$Q(t) = \sum_{i=1}^{n} q_i(t)$$

Our parameters a and b remain unchanged.

Introducing Time

Dynamical System

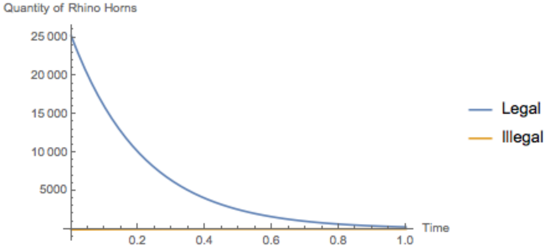
Since the quantities of every firm i now depends on time, we can obtain the following
time-derivatives of our two firms described in Section 3 (the legal and illegal markets).

$$\dot{q_1}(t) = \beta\left(\overline{q_1(t)} - q_1(t)\right)$$
$$\dot{q_2}(t) = \beta\left(\overline{q_2(t)} - q_2(t)\right)$$

where the dimension of β is the inverse of time.

Results

We now assume that we flood the market by releasing 100% of our stockpile initially.



Notice that the illegal market can be eradicated in a 1 year time period. The problem, however, is that in the process, the stock pile of the legal market also gets depleted. The problem with this is that this allows the poachers to return to poaching after a year. This can be improved or possibly prevented by letting our α depend on time, which will allow us to tactically release different percentages of our stockpile at different stages in time.

Conclusions

- Our simple models suggests that legalisation is a viable option for tackling the poaching problem.
- Furthermore, the entire stockpile need not be injected into the market (as the government intends to do).
- In the literature, however, there are more sophisticated models that suggest that legalisation may even intensify poaching - Fisher (2004), Abbott and van Kooten (2011), Bulte and Damania (2007), Bulte and van Kooten (1999).
- Further research is still needed to optimize our models and perhaps invalidate their results.

Further Work

- Both the dynamic and static games used linear price functions
- Variational conjectures were not determined from the games which made it impossible to find the global solution
- Dockner (1992) illustrated that for linear closed-loop games a unique and globally stable equilibrium exists.
- The objective is to use the build on the work of Docker, which was based on the work of Samuelson (1987), Driskill & McCafferty (1989) and Reynolds (1987) to find unique global solutions to applications in Rhino markets.